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BIRELATOR SPACES ARE NATURAL GENERALIZATIONS OF NOT ONLY BITOPOLOGICAL SPACES, BUT ALSO IDEAL TOPOLOGICAL SPACES

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Abstract. In 1962, W. J. Pervin proved that every topology $\mathcal{T}$ on a set X can be derived from the quasi-uniformity $\mathcal{U}$ on X generated by the preorder relations $R_A = A^2 \cup A^c \times X$ with $A \in \mathcal{T}$.

Thus, a quasi-uniform space $X(\mathcal{U})$ is a generalization of a topological space $X(\mathcal{T})$, and a bi-quasi-uniform space $X(\mathcal{U}, \mathcal{V})$ is a generalization of a bitopological space $X(\mathcal{P}, \mathcal{Q})$, studied first by J. C. Kelly in 1963.

Now, we shall show that a bi-quasi-uniform space $X(\mathcal{U}, \mathcal{V})$ is also a certain generalization of an ideal topological space $X(\mathcal{T}, \mathcal{J})$ studied first by K. Kuratowski in 1933.

Actually, instead of a bi-quasi-uniform space $X(\mathcal{U}, V)$, we shall use a birelator space $(X, Y)(\mathcal{R}, \mathcal{S})$, where X and Y are sets and $\mathcal{R}$ and $\mathcal{S}$ are relators (families of relations) on X to Y.

Much more general results could be achieved by using corelations (functions of $\mathcal{P}(X)$ to $\mathcal{P}(Y)$) instead relations on X to Y. However, a detailed theory of corelators has not been worked out yet.

Contents

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