UNIVERSITY OF DEBRECEN

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Árpád Száz

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CONTRA CONTINUITY PROPERTIES OF RELATIONS IN RELATOR SPACES

ÁRPÁD SZÁZ

ABSTRACT. In 1994, Julian Dontchev called a function f of one topological space X to another Y to be contra continuous if, for each open subset V of Y, the inverse image $f^{-1}[V]$ is a closed subset of X.

This seems to be a rather inconvenient continuity-like property. However, despite this, it has been intensively investigated by a surprisingly great number of prominent mathematicians.

Therefore, it seems reasonable to treat this notion in relator spaces having been developed by the present author and his PhD students. Namely, they provide the most convenient framework for continuity considerations.

Relator space, in a narrower sense, is an ordered pair $X(\mathcal{R}) = (X, \mathcal{R})$ consisting a set X and a family $\mathcal{R}$ of relations on X. Thus, it is a common generalization of ordered sets and uniform spaces.

In the relator space $X(\mathcal{R})$, we define some relations $\mathrm{Cl}_{\mathcal{R}}$, $\mathrm{Int}_{\mathcal{R}}$, $\mathrm{cl}_{\mathcal{R}}$, $\mathrm{int}_{\mathcal{R}}$, and families $\tau_{\mathcal{R}}$, $\tau\!\!\!-_{\mathcal{R}}$, $\mathcal{T}_{\mathcal{R}}$, $\mathcal{F}_{\mathcal{R}}$, $\mathcal{D}_{\mathcal{R}}$, $\mathcal{E}_{\mathcal{R}}$ such that, for instance, $A \in \mathrm{Int}_{\mathcal{R}}(B)$ if $R[A] \subseteq B$ for some $R \in \mathcal{R}$, and $A \in \mathcal{D}_{\mathcal{R}}$ if $X = \mathrm{cl}_{\mathcal{R}}(A)$.

Thus, instead of contra continuous functions, for instance, we investigate a relation F on one relator space $X(\mathcal{R})$ to another $Y(\mathcal{S})$ which reverses proximal interior in the sense that $A \in \mathrm{Int}_{\mathcal{R}}(B)$ implies $F[A] \in \mathrm{Cl}_{\mathcal{S}}\big(F[B]\big)$.

CONTENTS

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