# UNIVERSITY OF DEBRECEN

# VARIATION OF TILT ANGLES OF SOLAR ACTIVE REGIONS THROUGH THE 14$^{TH}$ SUNSPOT CYCLE

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**Abstract** Here the latitudinal (L) and time dependences of the tilt angles (γ) of clearly bipolar solar active regions (AR) have been studied through the $14^{th}$ sunspot cycle on the base of Greenwich Photo-Heliographic Results (GPHR). It was found that the yearly averaged tilt angles of given latitudinal (L) zones, in general within the error bars, don't show the effect of winding of magnetic fields except some deviations in the first and the last years of the solar cycle. Furthermore, on the base of observations and application of D'Silva and co-workers' analytical model it was found that the average rise rime of the emerging presumably Ω shape magnetic flux tubes from the bottom of the convective zone to the photosphere can be much shorter than it is thought in general. Moreover, also on the base of observations and analytical models of Parker, D'Silva and Choudhuri the estimation of rise time as the function of ARs' whole area (WA) in the case of areas larger than 400 millionths of the Sun's visible hemisphere was found to be in partial agreement with the theoretically expected behavior. That is the rise time increases as the WA or equivalently the strength of the magnetic field decreases. But the behavior of the rise time in the case of active regions with WA is smaller than 400 millionths does not follow this rule on the contrary, it decreases together with the WA.

**Keywords** solar cycle, active region, tilt angle, rise time, whole area

## 1. Introduction

It is believed that the origin of solar magnetic field is in the dynamo effect that operates in a stable layer at the base of the convection zone. According to dynamo models (Parker, 1955; Babcock, 1961; Leighton, 1964, 1969) the initial poloidal field of the Sun turns into toroidal because of differential rotation. Bipolar magnetic regions (BMRs) originate from these toroidal strands of magnetic flux, which may come out of this stable layer and would rise through the convection zone as an Ω-loop.

The raised flux tubes show Joy's law that is the BMRs are inclined to the latitudinal line, with the preceding (p) spots closer (+) and the following (f) spot away (-) from the equator as well as this tilt angle (γ) in general increases together with the absolute value of latitude (|L|) (Hale et al., 1919). Howard (1991) dealt with this question in detail for the first time and noted that there is no systematic variation of orientation angle with the cycle phase.

One of the solutions for the origin of the tilt is to take the Coriolis force into account that can twist the initially toroidal ascending flux loops (Schmidt, 1968) so that it finally emerges at the surface with a tilt (γ) to the local latitudinal line, This tilt (γ) depends on the latitude (L) and the rise time (t) of the emerging flux loop across the convective zone.

Although this tilt has been studied in several cases (Wang and Sheely, 1989, 1991; Howard, 1991, 1996, 1996b; Sivaraman, Gupta, and Howard, 1999, etc.) numerous questions have remained open.

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Thus, compared to the preceding results the present work is performed on the basis of different sunspot cycle, data sets of clearly oriented BMRs and evaluation procedure than others furthermore, it is a sequel of our earlier papers (Tóth and Gerlei, 2000; Tóth and Gerlei, 2004).

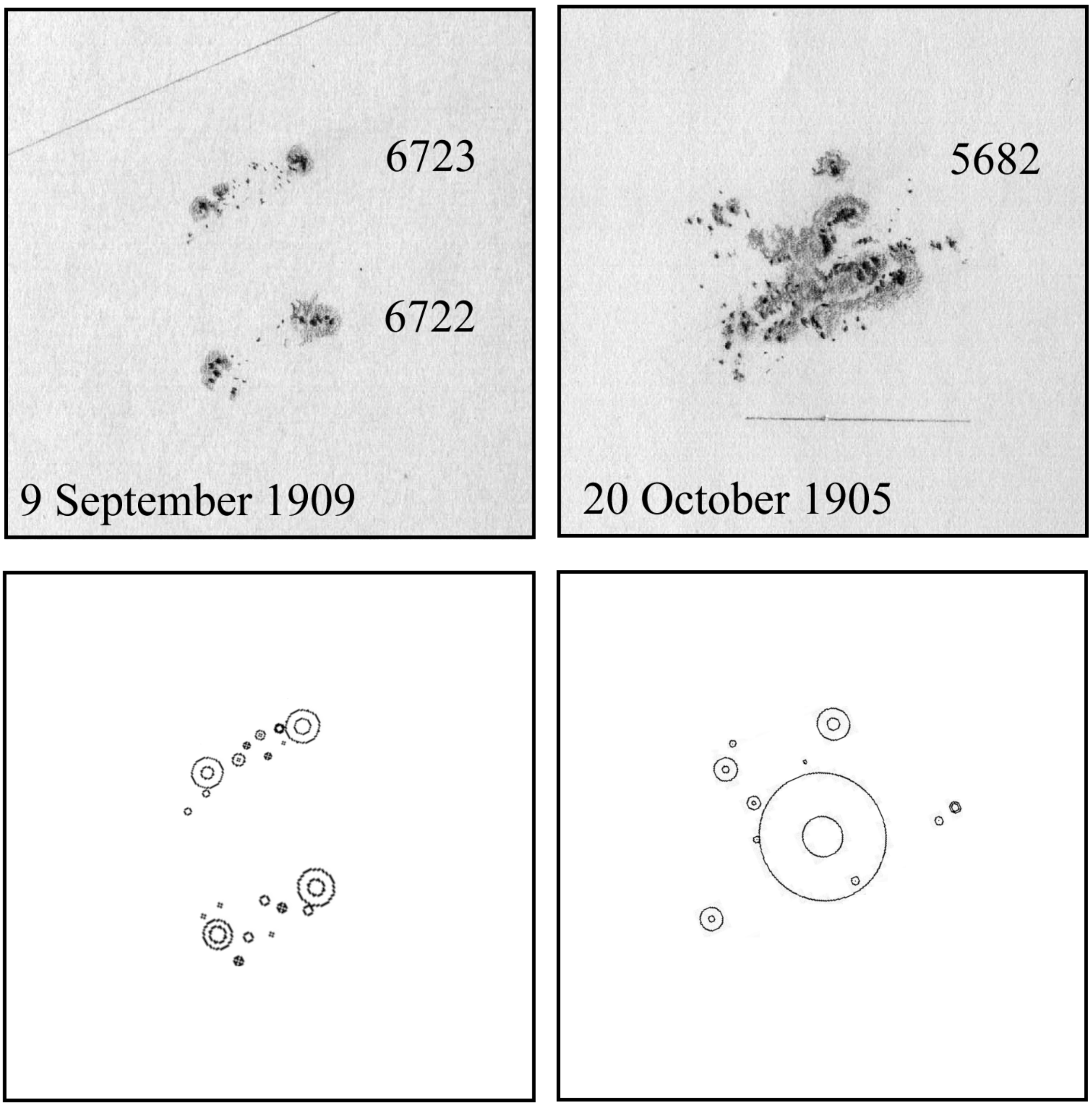


**Figure 1.** Examples for regular and clearly oriented active regions (left) that have been taken into account in the present work and the irregular ones (right) which were neglected, similarly to Tóth and Gerlei, (2004). The ARs are numbered on the base of GPHR. The upper two pictures are the parts of those excellent quality 6221 full disk solar photosphere drawings, which were made regularly from 1880 to 1919 in the Haynald Observatory Kalocsa, Hungary (Tóth, Mező, and Gerlei, 2002). The lower two pictures are reconstructions of the given ARs that were made on the basis of position and area data of GPHR catalogue.

## 2. Method of Investigation

In the present investigation the data of Greenwich Photo-Heliographic Results (GPHR) concerning the 14th solar cycle from 1901 to 1913 was applied. In the original printed editions of GPHR the heliographic latitudes (L) and longitudes (ϕ) of sunspots are given with accuracy of 0.1 degree while the areas of them are expressed in millionth of the Sun's visible hemisphere, which are approximately equal with the errors of them (Christie, 1909; Newton, 1958; Kovács, 1987; Gerlei 1987).

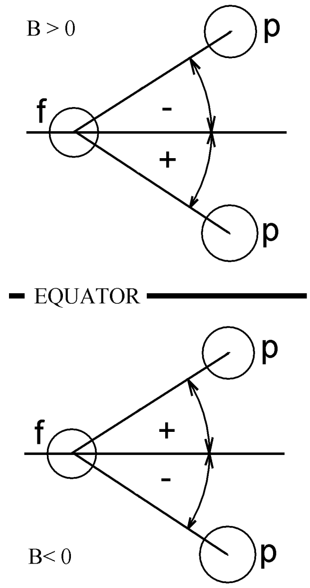


**Figure 2.** The interpretation of the sign of the tilt angle of active regions.

Since there was no magnetic field information available for the investigated sunspot groups, that fact in general has no significant effect on the visible light observations based tilt angle measurements (Wang et al., 2015), in the present cases an active region (AR) could have had been identified as BMR if the p- and f-parts of it were separable well. This circumstance led to the selection of the visually identifiable BMRs, which resulted additional work but partly was fortunate since the complicated ARs, which have no clear alignment, were not taken into account. Taking into consideration of these 'irregular' groups would just have led to larger deviations of the measured tilt angles (γ) (Figure 1).
Because of large distortion appears close to the solar limb only those ARs were taken into account that longitudinal (ϕ) co-ordinates were not farther from the central meridian than ±60°. The above-mentioned selections of the suitable ARs were made on the base of the visualized data of GPHR catalogues and on the digitalized photosphere observations of the Haynald Observatory (Figure 1) (Tóth, Mező, and Gerlei, 2002). This selection resulted in 684 pieces of different numbered ARs (from the total 2074 of the 14th solar cycle in the GPHR catalogue), which were observed altogether in 3754 cases since most of them were visible for several days.

The tilt angle ($\gamma$) calculation of selected ARs was based on the position data of GPHR. This angle is by convention positive for ARs that p-spots are equatorward and negative if poleward (Sivaraman, Gupta, and Howard, 1999) (Figure 2). The tilt angle was calculated as the bend of a straight line to the local latitudinal line fitted by area weighted least-squares method to the individual spots of the given AR considering Howard's (1991) latitudinal correction ($\tan \gamma_{cor} = \tan \gamma / \cos |L|$, where $\gamma$ is the tilt angle and L is the latitude) as well. Furthermore, in the course of error calculations we applied standard deviations and mean error propagation.

## 3. The Tilt Angle as the Function of Time and Latitude

As it was stated in the introduction if the tilt ($\gamma$) is caused only by the winding of magnetic field it should then decrease at a given latitude (L) as the field lines are stretched further with the progress of the solar cycle (Figure 3).

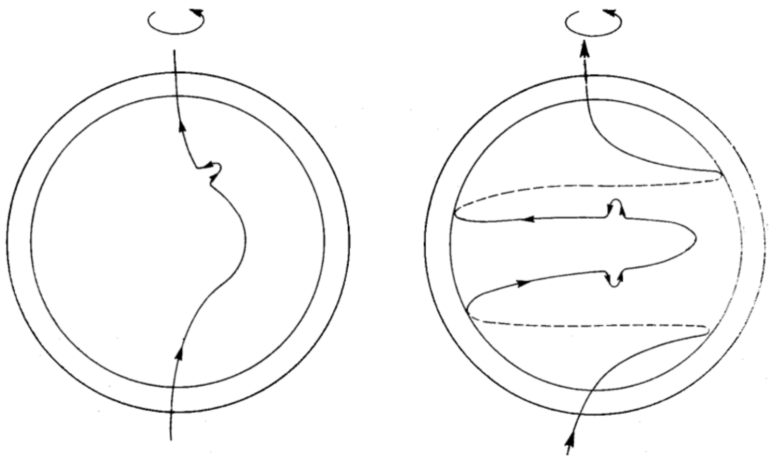

**Figure 3.** The presumable winding of the solar magnetic field during the solar cycle and the emerging $\Omega$-loops.

Hoping to get some evidence of this, we continued Howard's (1991) work and investigated the averaged tilt angles over one year intervals of time and 5° intervals of latitude through the 14$^{th}$ sunspot cycle (Table 1, Figure 4). In spite of the fact that there are some incomprehensible differences in the tilt angles around the first and last years of the solar cycle that are larger than their error bars in most parts of the cycle there are no significant changes in the yearly averaged tilt angles at different latitudes (|L|). However, these differences in the first and last years may mean that the winding of the initial poloidal field into closely toroidal is happen within the last and first years of the sunspot cycle that phenomena would have been worthy to investigate in detail with better statistics, higher time and angular resolution. In consequence of this we can strengthen that the supposed winding of the magnetic lines during the solar cycle has no substantial role in the formation of the tilt angle of active regions, except in the first and last years of the solar cycle which result is in agreement with an assumed nearly toroidal field at the tachocline in general and favorable in terms of the Coriolis force hypothesis.

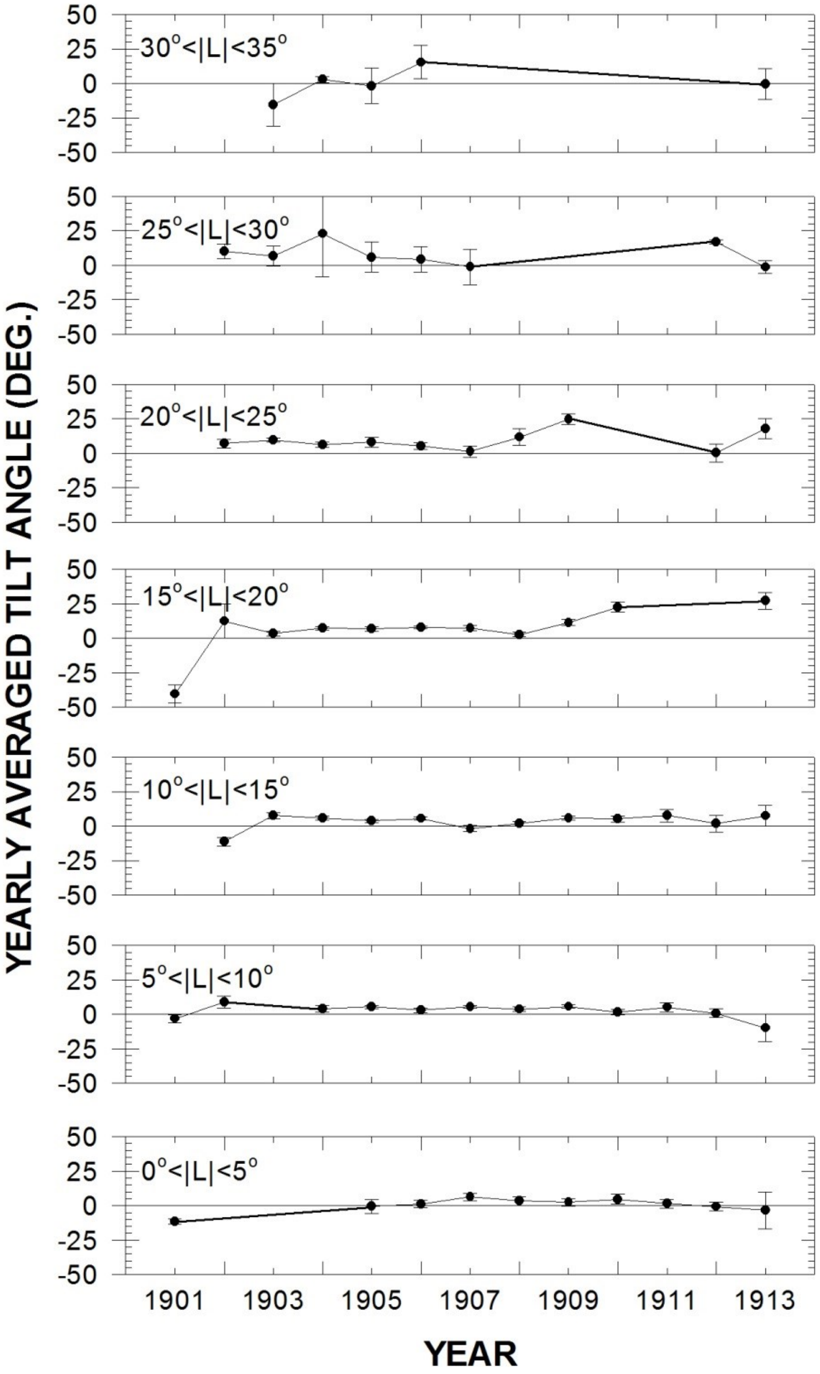


**Figure 4.** The averaged tilt angles of regular and clearly oriented active regions over a year- and 5° intervals of the absolute value of latitudes (|L|). The minimum solar activities were at 1901.7 and 1913.6 and the maximum was at 1907.0 (see Table 1)

**Table 1.** The averaged tilt angles (γ) of regular and clearly oriented active regions (in degree) over a year- and 5° intervals of |L| latitudes. The minimum solar activities were at 1901.7 and 1913.6 and the maximum was at 1907.0 (see Figure 4)

| Latitude zone | Year | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1901 | 1902 | 1903 | 1904 | 1905 | 1906 | 1907 | 1908 | 1909 | 1910 | 1911 | 1912 | 1913 |
| 30°≤\|L\|<35° | | | -15.6±15.6 | 2.9±2.1 | -1.7±12.9 | 15.4±12.2 | | | | | | | -0.3±11.3 |
| 25°≤\|L\|<30° | | 10.3±5.4 | 6.9±7.1 | 23±31 | 6±11 | 4.5±9.2 | -1.1±12.9 | | | | | 17±1.6 | -1.2±4.6 |
| 20°≤\|L\|<25° | | 7.1±3.2 | 9.7±1.4 | 6.2±2.1 | 7.9±3.8 | 5.2±2.5 | 1.2±4 | 11.8±6 | 24.5±3.9 | | | 0.3±6.7 | 17.8±7.2 |
| 15°≤\|L\|<20° | -40.4±6.7 | 12.5±12.5 | 3.5±1.7 | 7.4±1.1 | 6.9±1.6 | 7.9±1.1 | 7.5±2 | 2.7±1.8 | 11.5±2.1 | 22.6±3.6 | | | 27.2±5.9 |
| 10°≤\|L\|<15° | | -11.4±3 | 7.7±2.3 | 5.8±1.6 | 3.8±1.2 | 5.6±1.3 | -1.8±2.1 | 1.9±1.6 | 5.9±1.5 | 5.3±2,1 | 7.6±4.6 | 1.7±5.9 | 7.5±7.5 |
| 5°≤\|L\|<10° | -3.1±2.8 | 9.0±4.5 | | 4.1±2.5 | 5.4±1.2 | 2.9±1.8 | 5.6±1.1 | 3.8±1.6 | 5.6±1.4 | 1.6±2.2 | 3.4±39 | 0.8±3.2 | -9.8±9.8 |
| 0°≤\|L\|<5° | -11.7±1.8 | | | | -0.6±4.9 | 1.2±2.7 | 6.3±2.7 | 3.4±3.2 | 2.3±2.9 | 4.5±3.7 | 1.4±3.2 | -0.8±3.2 | -3.6±13.4 |

**Table 2.** The averaged tilt angles (γ) of regular and clearly oriented active regions (in degree) over the 14$^{th}$ sunspot cycle as the function of 5° intervals of |L| latitudes and given whole area (WA) active regions. The WA is given in millionths of the Sun's visible hemisphere. The minimum solar activities were at 1901.7 and 1913.6 and the maximum was at 1907.0 (see Figures 5, 7)

| Latitude zone | Averaged tilt angle (γ) over the given intervals of latitude | | | | | | |
|---|---|---|---|---|---|---|---|
| | WHOLE (0≤WA) | 0≤WA<100 | 100≤WA<200 | 200≤WA<400 | 400≤WA<600 | 600≤WA<1000 | 1000≤WA |
| 0°≤\|L\|<5° | 2.3± 1.2 | 3.5± 1.7 | 0.3± 2.2 | -0.6± 2.5 | 4.8± 2.7 | 2.0± 2.2 | 2.5± 2.3 |
| 5°≤\|L\|<10° | 4.2± 0.6 | 4.9± 1.0 | 3.9± 1.3 | 4.9± 1.2 | 3.1± 1.4 | 1.3± 1.3 | 5.7± 1.7 |
| 10°≤\|L\|<15° | 4.3± 0.6 | 3.5± 1.0 | 3.2± 1.3 | 7.1± 1.2 | 7.9± 1.4 | 2.5± 1.6 | -3.0± 3.4 |
| 15°≤\|L\|<20° | 7.4± 0.6 | 7.7± 0.9 | 7.2± 1.3 | 6.5± 1.4 | 6.4± 1.5 | 9.2± 1.6 | 6.1± 7.2 |
| 20°≤\|L\|<25° | 7.5± 1.0 | 8.1± 1.5 | 5.2± 2.7 | 6.9± 1.9 | 15.9± 2.5 | 10.3± 4.4 | -2.0± 1.9 |
| 25°≤\|L\|<30° | 7.1± 4.0 | 2.0± 4.9 | 34.8± 10.2 | 14.8± 3.4 | 11.6± 14.6 | 3.1± 6.8 | |
| 30°≤\|L\|<35° | 1.9± 4.3 | 1.4± 6.3 | 3.0±3.2 | 2.8± 2.8 | | | |

## 4. Joy's Law and the Possible Role of the Coriolis Force

A lot of information is obtainable from the tilt angle ($\gamma$) distribution of ARs as the function of latitude (L), which in general form is the "Joy's Law" (Hale et al., 1919) that presumably appears in consequence of acting the Coriolis force on the emerging flux tube (Schmidt, 1968). Our relevant results (Table 2. and Figure 5.) agree well with the Kodaikanal and the Mount Wilson data (Howard, 1991; Sivaraman, Gupta, and Howard, 1999). The only substantial difference is in the 30°-35° latitude zone because of the relatively small number of the investigated ARs in our case there, but it corresponds to others within the error bars.

It is worthy to compare the present results of latitudinal (|L|) tilt angle ($\gamma$) distribution (Table 2, Figure 5.) with the results of Tóth and Gerlei (2000) (see Table 2 and Figure 2. there) where the tilt angle ($\gamma$) was calculated in different way than in the present paper (see Section 2). In Tóth and Gerlei (2000) at first the whole area (WA) weighted positions of p and f parts of the given AR were calculated and then the tangent of the orientation angles while here the tilt angle ($\gamma$) was calculated directly as the bend of a straight line to the local latitudinal (L) line fitted by the whole area (WA) weighted least-squares method to the individual spots of the given AR. In both cases the Howard's (1991) type latitudinal correction ($\tan \gamma_{cor} = \tan \gamma / \cos |L|$, where $\gamma$ is the tilt angle and L is the latitude) were applied in the same way.

There are several numerical models and calculations, which attempt to describe the behavior of the emerging flux tube. However, because of the large number of free parameters in these calculations the comparison of them with observations is complicated. The solution for this problem is the reduction of the numerous unknown free parameters or the application of a rough but still useable analytical formula that simplifies strongly but may qualitatively be applicable for the measured data.

In accordance with the above mentioned problems we have chosen the last case that applies an analytic model developed by D'Silva and Choudhuri, (1993) for the role of Coriolis force in the formation of the latitudinal tilt angle distribution of BMRs. The final equation of this theory in our case is:

$$\gamma = \tan^{-1}(\tan(\Omega t) \sin(L)) \qquad (1)$$

where $\gamma$ [rad] is the tilt angle of BMR, $\Omega = 2.8 * 10^{-6}$ [$sec^{-1}$] is the average rotational frequency of the Sun, t [sec] is the rise time of the flux tube and L [rad] is the latitude.

The rise time, according to Parker (1975) and D'Silva and Choudhuri (1993), is possible to be written as:

$$t = (R_{\odot}/3B_m) * \sqrt{(C_d * H * 4 * \rho/\sigma)} \qquad (2)$$

where $R_{\odot}$ is the solar radius, $B_m$ is the magnetic field in the flux tube, $C_d$ is the drag coefficient, H is the scale height, $\rho$ is the density inside the flux tube and $\sigma$ is the radius of the flux tube, which values are averaged in space and time.

The selection of this simplified theoretical description is strengthened by the fact that in the case of similar initial parameters this model within the observed error bars leads to the same results as the more difficult and presumably realistic numerical calculations (D'Silva and Choudhuri, 1993).

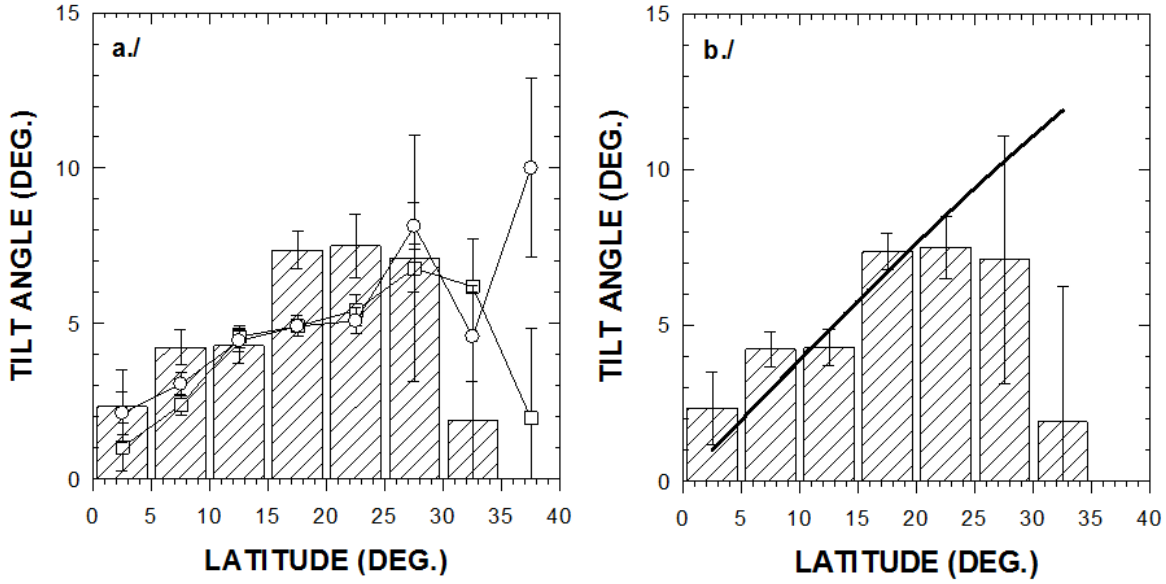


**Figure 5.** The distribution of tilt angles (γ) as the function of latitude (|L|) averaged over 5° intervals. (a) The striped columns show our results that are based on the selected regular and clearly oriented active regions (stated above). The open circles represent the Kodaikanal while squares the Mount Wilson data (based on Sivaraman, Gupta, and Howard, 1999). (b) The striped columns show our result again and the solid line is the error weighted least-squares fit of equation 1. to our data (see Table 2).

So we fitted the Equation (1) to our data by the error weighted least-squares method (Table 2, Figure 5.) It resulted in the fitting parameter as $\Omega t = 0.37 \pm 0.03$ that in consequence of the periodicity of the function 'tan$^{-1}$' is formed to $\Omega t = 0.37 \pm 0.03 + k\pi$ where k is (k=0, 1, 2, etc.) integer.

In the cases k>0 the flux tubes do not only twist (k=0) but they also make one or more (k pieces) turns during their emerging. This kind of behavior, which would lead easily to the breaking of the flux-loops, is not impossible since Lopez Fuentes et al. (2000) observed it.

But apart from this the general conception is that the emerging flux tubes are only twist (case of k= 0). Then on the base of our result ($\Omega t = 0.37 \pm 0.03$) the rise time (t) of an average flux tube from the basis of the convection zone is given as $1.3 \pm 0.1*10^5$ second that is just 1.5 ± 0.1 days. And the average rise velocity through the convection zone is 1.8 ± 0.1 km/s, since the depth of the convection zone is about $R_{\odot}/3 = 2.32*10^5$ km. Though this time is shorter and this average velocity is larger with orders of magnitudes than it was predicted by different theories before, the last one is basically the same as it was measured close to the solar surface in several other cases (Bray, Loughhead, and Durrant, 1984; Kosovichev, 2000, etc.).

The above results may mean three different things:

- It would mean that the rise time from the bottom of the connective zone to the surface is probably much shorter than the generally accepted several months to years long interval.
- There is a tiny possibility of that when an Ω-loop after a short rise time reaches its Joy's angle ($\gamma_J$) then disconnects still close to the bottom of the convection zone and emerges further slowly with its tilt ($\gamma_J$) without the restriction of its lower part.
- Also with tiny probability, but it could also mean that the available general conceptions concerning the origin and the following rise up of the flux tubes are no longer acceptable

**5. The Rise Time of Emerging Flux Tube as the Function of Active Region Area**

As it is generally accepted and visible in the Equation (2) as well, the rise time (t) is presumably in inverse proportional relation with the magnetic flux ($\Phi=B_m\pi\sigma^2$) of the flux tube (Parker, 1975; D'Silva and Choudhuri, 1993). The control of this assumption in a way is possible since the rise time (t) is determinable from the fitting of Equation (1) to the latitudinal (|L|) tilt angle ($\gamma$) distributions of active regions which belong to the strength of different magnetic fields ($B_m$). Although, we have no magnetic field information for the studied 14$^{th}$ sunspot cycle but there is a well-known role which states that the magnetic flux ($\Phi$) of ARs is proportional with their area (Tian et al., 1999).

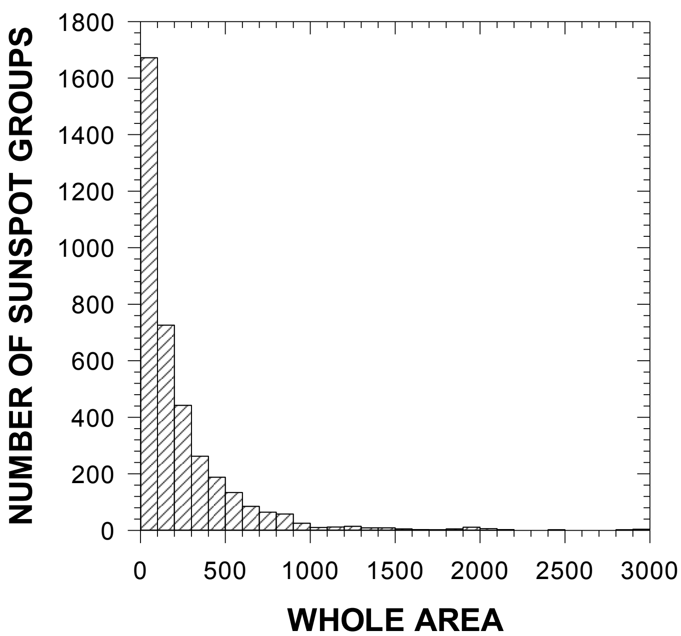


**Figure 6.** The whole area distribution of the selected regular and clearly oriented active regions

Therefore, at first we determined the sunspot group whole area (WA) distribution of the studied active regions (Figure 6) of the $14^{th}$ sunspot cycle and gathered them into six different whole area (WA) groups (WA= 0-99, 100-199, 200-399, 400-599, 600-999, >=1000 in millionths of the Sun's visible hemisphere). The latitudinal (|L|) tilt angle (γ) distributions of them are visible in the Table 2 and Figure 7. Henceforth, the Equation (1) was fitted to each of the above mentioned groups of active regions (Table 2, Figure 7).

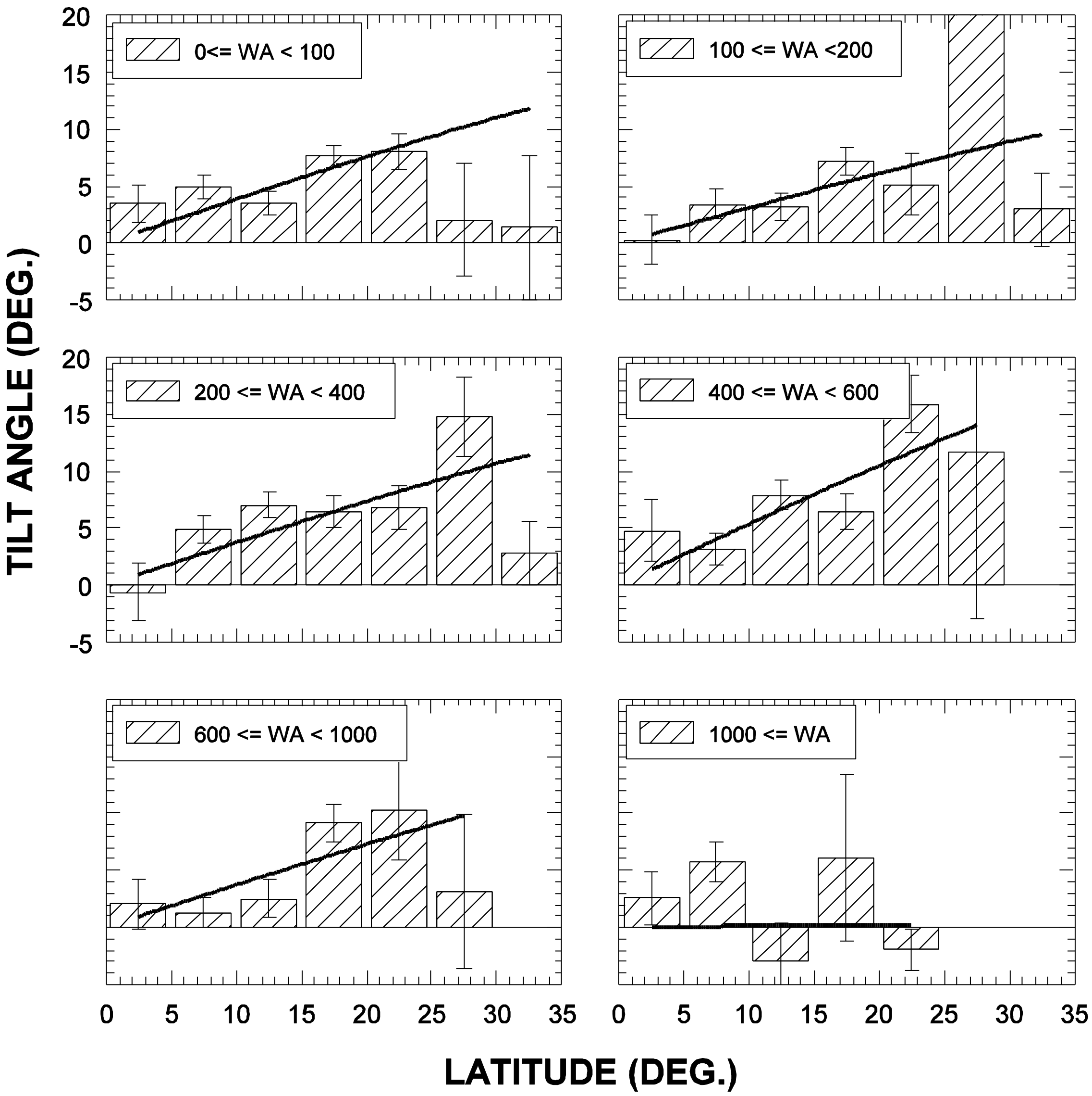


**Figure 7.** The distribution of tilt angles (γ) of regular and clearly oriented active regions averaged over 5° intervals of latitude (|L|) in the cases of different size whole areas (WA) active regions (striped columns). The solid line is the error weighted least-squares fit of Equation (1) to the tilt angle distributions (see Table 2).

**Table 3.** The Ωt parameters resulted from the fitting of equation 1 to the latitudinal (|L|) tilt angle (γ) distributions (see Table 2 and Figure 7) of different whole area (WA) groups of ARs (see Figures 7, 8) for the 14th sunspot cycle. The WA is given in millionths of the Sun's visible hemisphere.

| WA groups | 0 <= WA<100 | 100 <= WA<200 | 200 <= WA<400 | 400 <= WA<600 | 600 <= WA<1000 | 1000<= WA |
|---|---|---|---|---|---|---|
| Average WA of the above given WA groups | 46.2± 0.6 | 146± 1 | 285± 2 | 492± 3 | 759± 7 | 1612± 51 |
| Ωt | 0.37± 0.05 | 0.30± 0.07 | 0.36± 0.07 | 0.50± 0.06 | 0.36± 0.07 | 0.01± 0.14 |

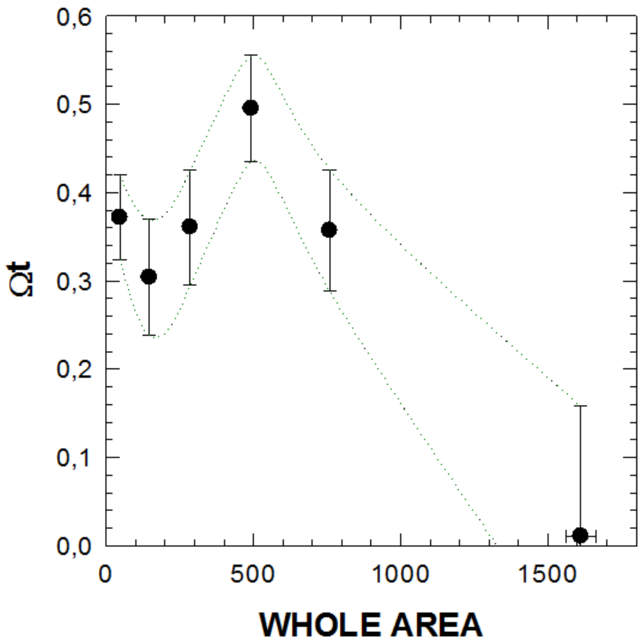


**Figure 8.** The Ωt parameter as the function of whole area (WA) of regular and clearly oriented active regions. The dotted line is just for guiding the eye (see Table 3).

It resulted in the 'Ωt' parameters as the function of the average whole area (WA) of the given ARs (Table 3, Figure 8), or as it was stated before indirectly as the function of their magnetic flux (Φ). The outcomes are remarkable.

In cases when the whole area of active regions is larger than 400 millionths the result is in agreement with the generally expected by theories; that is the rise time increases as the whole area or equivalently the magnetic flux (Φ) decreases (Equation 2). But as it is visible the behavior of the rise time in the case of active regions with area smaller than 400 millionths does not follow this rule on the contrary, it shows that the rise time decreases as the whole area decreases (Table 3, Figure 8). And it would have a particular significance since more than the 80% of the selected active regions have smaller whole area than 400 millionths (figure 6).

## 6. Conclusions

First of all, on the basis of this work we can claim that the GPHR catalogue together with other historical datasets (e.g. Tóth, Mező, and Gerlei, 2002) can be used nowadays as well.

Our investigation is basically in agreement with others and shows that in yearly average there is no clear time dependency of the tilt angle in the investigated latitude zones along the main part of the solar cycle accept some differences around the beginning and the end of the cycle that is worthy for further investigation. Consequently the assumed process that is the winding of initial poloidal field of the Sun into toroidal may has no substantial role in the formation of the tilt angle of active regions during the major part of the solar cycle.

Taking the Coriolis force, which is presumably responsible for Joy’s law, into account provided us with a possibility to give estimation for the typical average rise time of magnetic flux tube, which resulted in being much smaller than the expected value. It would mean that the Ω-loop rises up much faster than it is imagined or that it disconnects from its lower parts still close to the bottom of the convective zone or that the phenomenon is completely different than it is accepted in general.

And finally, the further investigation of Joy’s law as the function of active region area permit us to investigate the dependence of the rise time of the presumably anchored Ω-loops as the function of their magnetic flux since, there is a direct observations based connection between the area of active regions and their magnetic field. In cases when the active region areas are larger than 400 millionth of the Sun’s visible hemisphere our results are in agreement with the general ideas. That is the rise time increases as the area decreases. But in cases of smaller area than 400 millionth the rise time shows opposite behavior.

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