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# Relationships between inclusions for relations and inequalities for corelations

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**Preprints No. 422**
(Technical Reports No. 2017/4)

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**2017**

# RELATIONSHIPS BETWEEN INCLUSIONS FOR RELATIONS AND INEQUALITIES FOR CORELATIONS

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Abstract. A function $U$ on one power set $\mathcal{P}(X)$ to another $\mathcal{P}(Y)$ will be briefly called a corelation on $X$ to $Y$. Thus, complementation and closure (interior) operations on $X$ are corelations on $X$.

Moreover, for any two corelations $U$ and $V$ on $X$ to $Y$, we shall write $U \leq V$ if $U(A) \subseteq V(A)$ for all $A \subseteq X$. Thus, the family of all corelations on $X$ to $Y$ also forms a complete poset (partially ordered set).

Formerly, we have established a partial Galois connection $(\triangleright, \triangleleft)$ between relations and corelations. Now, by using this, we shall establish some further relationships between inclusions for relations and inequalities for corelations.

For instance, for some very particular corelations $U$ and $V$ on $X$ to $Y$, with $U^{\triangleleft} \leq V^{\triangleleft}$, we shall prove the existence of an union-preserving corelation $\Phi$ on $X$ to $Y$ which separates $U$ and $V$ in the sense that $U \leq \Phi \leq V$.

## 1. Introduction

In our former paper [17], a function $U$ on one power set $\mathcal{P}(X)$ to another $\mathcal{P}(Y)$ has been briefly called a corelation on $X$ to $Y$. Thus, complementation and closure (interior) operations on $X$ are corelations on $X$.

If $R$ is a relation on $X$ to $Y$, i.e., $R \subseteq X \times Y$, then the function $R^{\triangleright}$, defined by $R^{\triangleright}(A) = R[A] = \bigcup_{x \in A} R(x)$ for all $A \subseteq X$, can be easily seen to be a union-preserving corelation on $X$ to $Y$ which may be identified with $R$.

Conversely, if $U$ is a corelation on $X$ to $Y$, then we may naturally define a relation $U^{\triangleleft}$ on $X$ to $Y$ such that $U^{\triangleleft}(x) = U(\{x\})$ for all $x \in X$. Moreover, for the corelation $U$, we may also naturally write $U^{\circ} = \left(U^{\triangleleft}\right)^{\triangleright}$.

Namely, for any two corelations $U$ and $V$ on $X$ to $Y$, we may also naturally write $U \leq V$ if $U(A) \subseteq V(A)$ for all $A \subseteq X$. Thus, the family of all corelations on $X$ to $Y$ also forms a complete poset (partially ordered set).

Moreover, we can show that the functions $\triangleright$ and $\triangleleft$ establish a partial Galois connection in the sense that, for an arbitrary relation $R$ and a quasi-increasing corelation $U$ on $X$ to $Y$, we have $R^{\triangleright} \leq U$ if and only if $R \subseteq U^{\triangleright}$.

Now, a corelation $U$ on $X$ to $Y$ may be briefly called open (quasi-increasing) if $U \leq U^{\circ}$ $\left(U^{\circ} \leq U\right)$. Moreover, we can easily see that $U$ is union-preserving if and only if $U = U^{\circ}$. That is, $U$ is both open and quasi-increasing.

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2010 *Mathematics Subject Classification.* 06A15, 54C60.

*Key words and phrases.* Relations, setfunctions, Galois connections.

The work of the author has been supported by the Hungarian Scientific Research Fund (OTKA) Grant K-111651.