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Corelations are more powerful tools than relations

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CORELATIONS ARE MORE POWERFUL TOOLS THAN RELATIONS

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To the Memory of my younger brother Géza Száz

ABSTRACT. A subset R of a product set $X \times Y$ is called a relation on X to Y. While, a function U of one power set $\mathcal{P}(X)$ to another $\mathcal{P}(Y)$ is called a corelation on X to Y. Moreover, families $\mathcal{R}$ and $\mathcal{U}$ of relations and corelations on X to Y are called relators and corelators on X to Y, respectively.

Relators on X has been proved to be more powerful tools than generalized proximities, closures, topologies, filters and convergences on X. Now, we shall show that corelators on X to Y are more powerful tools than relators on X to Y. Therefore, corelators have to be studied before relators.

If $\mathcal{U}$ is a corelation on X to Y, then instead of the notation $\Subset_{\mathcal{U}}$ of Yu. M. Smirnov, for any $A \subseteq X$ and $B \subseteq Y$, we shall write $A \in \operatorname{Int}_{\mathcal{U}}(B)$ if there exists $U \in \mathcal{U}$ such that $U(A) \subseteq B$. Namely, thus we may also naturally write $\operatorname{Cl}_{\mathcal{U}}(B) = \mathcal{P}(X) \setminus \operatorname{Int}_{\mathcal{U}}(Y \setminus B)$, and $x \in \operatorname{int}_{\mathcal{U}}(B)$ if $\{x\} \in \operatorname{Int}_{\mathcal{U}}(B)$.

Moreover, we can also note note that $\operatorname{Int}_{\mathcal{U}}$ is a relation on $\mathcal{P}(Y)$ to $\mathcal{P}(X)$ such that $\operatorname{Int}_{\mathcal{U}} = \bigcup_{U \in \mathcal{U}} \operatorname{Int}_U$ with $\operatorname{Int}_U = \operatorname{Int}_{\{U\}}$. Therefore, the properties of the relation $\operatorname{Int}_{\mathcal{U}}$ can be immediately derived from those of the relations Int_U. This shows that corelations have to be studied before corelators.

For this, following the ideas of U. Höhle and T. Kubiak and the notations of B. A. Davey and H. A. Priestly, for any relation R and corelation U on X to Y, we define a corelation $R^{\triangleright}$ and a relation $U^{\triangleleft}$ on X to Y such that $R^{\triangleright}(A) = R[A]$ and $U^{\triangleleft}(x) = U(\{x\})$ for all $A \subseteq X$ and $x \in X$.

Here, for any two corelations U and V on X to Y, we may naturally write $U \leq V$ if $U(A) \subseteq V(A)$ for all $A \subseteq X$. Thus, the maps $\triangleright$ and $\triangleleft$ establish a Galois connection between relations and quasi-increasing corelations on X to Y such that $R^{\triangleright\triangleleft} = R$, but $U^{\triangleleft\triangleright} = U$ if and only if U is union-preserving.

Now, for any two corelations U on X to Y and V on Y to Z, we may also naturally define $U^{\circ} = U^{\triangleleft\triangleright}$, $U^{\mathbf{-1}} = U^{\triangleright-1\triangleright}$ and $V \bullet U = \big(V^{\triangleleft} \circ U^{\triangleleft}\big)^{\triangleright}$. Moreover, for instance, for any relator $\mathcal{R}$ on X to Y, we may also naturally define $\operatorname{Int}_{\mathcal{R}} = \operatorname{Int}_{\mathcal{R}^{\triangleright}}$ and $\operatorname{int}_{\mathcal{R}} = \operatorname{int}_{\mathcal{R}^{\triangleright}}$ with $\mathcal{R}^{\triangleright} = \big\{ R^{\triangleright} : \ R \in \mathcal{R} \big\}$.

Thus, in general $\operatorname{Int}_{\mathcal{U}}$ is a more general relation than $\operatorname{Int}_{\mathcal{R}}$. However, for instance, we already have $\operatorname{int}_{\mathcal{U}} = \operatorname{int}_{\mathcal{U}^{\triangleleft}}$. Therefore, our former results on the relation $\operatorname{int}_{\mathcal{R}}$ and the families $\mathcal{E}_{\mathcal{R}} = \big\{ B \subseteq Y : \ \operatorname{int}_{\mathcal{R}}(B) \neq \emptyset \big\}$ and $\mathcal{T}_{\mathcal{R}} = \big\{ A \subseteq X : \ A \subseteq \operatorname{int}_{\mathcal{R}}(A) \big\}$, whenever $X = Y$, will not be generalized.

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